

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS  
SPECIALIST  
UNIT 1

**SOLUTIONS**

Section One:  
Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 50 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 92 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

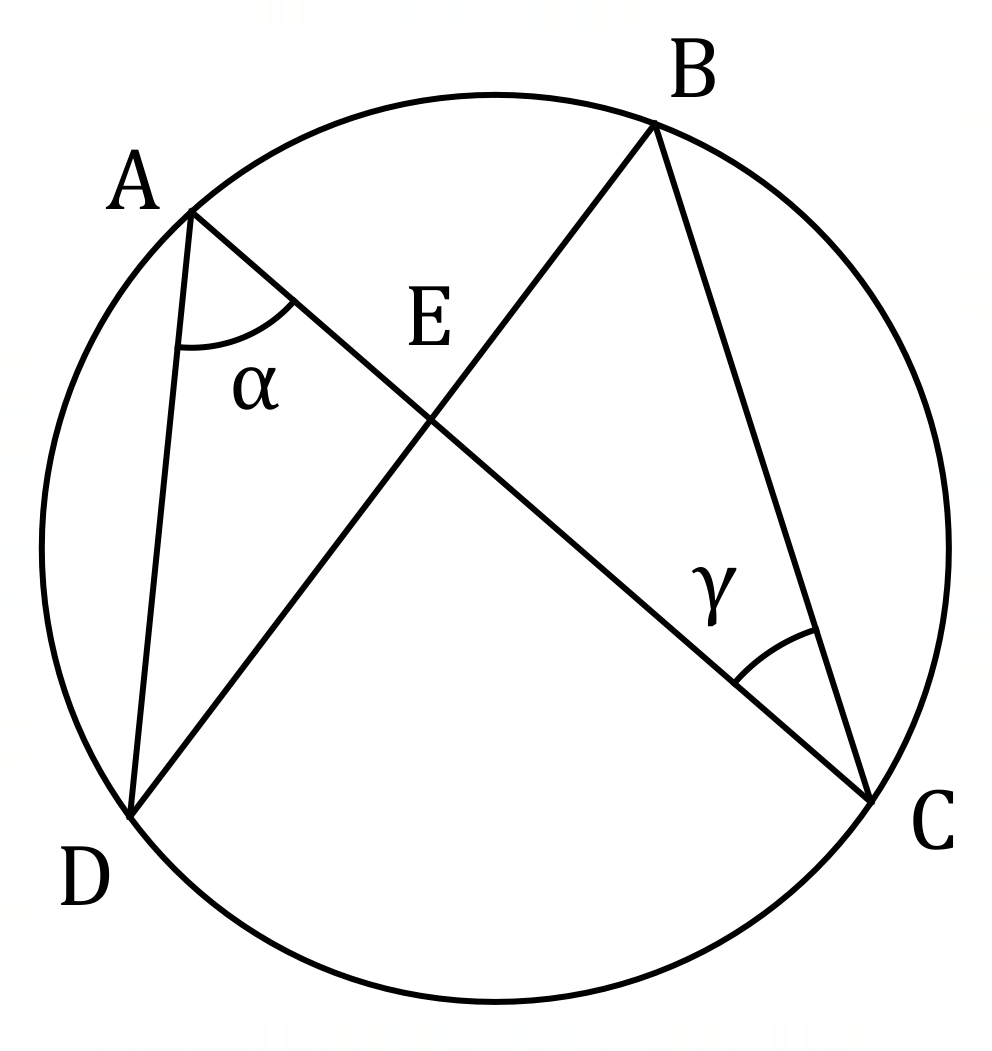
Section One: Calculator-free 35% (50 Marks)

This section has**eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (5 marks)

(a) In the circle shown, chords and intersect at , and .  
  
Determine the size of angles and . (2 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of  ✓ value of |

(b) In the circle below, is a cyclic quadrilateral and is a tangent to the circle at .  
  
Given that and , determine . (3 marks)

<EFOFEX>
id:fxd{d2085924-1b3c-41a1-bb79-78f9df548c35}

FXData:
</EFOFEX>

|  |
| --- |
| **Solution** |
| Alternate segment:  Angle on a line:  Opposite angles: |
| **Specific behaviours** |
| ✓ states or  ü calculates  ü calculates |

Question 2 (6 marks)

Points and lie on the circumference of a circle so that the chords and intersect at point .

(a) Sketch a diagram to show triangle and triangle and prove that they are similar.

(4 marks)

|  |
| --- |
| **Solution** |
| <EFOFEX> id:fxd{c8b9b0ae-44be-41e0-b489-979f7b8bd589}  FXData: </EFOFEX>  (angles stand on same arc)  (vertically opposite angles)  Hence triangles are similar as three pairs of equal angles. |
| **Specific behaviours** |
| ✓ neat, labelled sketch  ✓ one pair of angles, with reason  ✓ second pair of angles, with reason  ✓ summary, using AAA reasoning |

(b) In the case when the lengths of and are cm, cm and cm respectively, determine the length of . (2 marks)

|  |
| --- |
| **Solution** |
| Using intersecting chord theorem or similar triangles from (a): |
| **Specific behaviours** |
| ✓ indicates appropriate method  ü calculates length |

Question 3 (6 marks)

Let and .

(a) Determine . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates multiple  ü calculates sum  ✓ calculates magnitude |

(b) Determine the vectors and given that and .

(3 marks)

|  |
| --- |
| **Solution** |
| Adding equations to eliminate : |
| **Specific behaviours** |
| ✓ eliminates one vector  ü calculates  ü calculates |

Question 4 (4 marks)

<EFOFEX>
id:fxd{c96a36c0-ce7d-43db-b4d9-28ecbacbafa7}

FXData:
</EFOFEX>In the diagram, is a tangent at to  
the circle with centre , intersects  
the circle at and chord is parallel  
to tangent .  
  
Determine the size of when  
the size of .  
  
Justify your answer.

|  |
| --- |
| **Solution** |
| Using angle between radius and tangent property:  Using angle at centre and circumference on same arc property:  Using alternate angle property: |
| **Specific behaviours** |
| ✓ angle at centre  ü angle on circumference  ü justifies both above properties  ü correct angle |

Question 5 (8 marks)

(a) Determine the vector(s) that are parallel to and have the same magnitude  
as . (4 marks)

|  |
| --- |
| **Solution** |
| Magnitudes:  Unit vector in required direction:  Hence vectors are: |
| **Specific behaviours** |
| ✓ both magnitudes  ü unit vector in required direction  ü one required vector  ü second vector in opposite direction |

(b) Two vectors are and , where is a constant.  
  
Determine the value(s) of so that the vectors are perpendicular. (4 marks)

|  |
| --- |
| **Solution** |
| Require scalar product to be zero: |
| **Specific behaviours** |
| ✓ equates scalar product to zero  ü calculates scalar product and expands  ü simplifies and factorises  ü states both values |

Question 6 (6 marks)

<EFOFEX>
id:fxd{7a791319-a0d8-45a3-9d2b-f2944fa2b72b}

FXData:
</EFOFEX>The diagram shows parallelogram .

A theorem states that the sum of the squares of  
the lengths of the diagonals of a parallelogram  
is equal to the sum of the squares of the lengths  
of its sides.

(a) Complete the following expression of the theorem using vector notation:

|  |
| --- |
| **Solution** |
| *Allow or or , etc.* |
| **Specific behaviours** |
| ✓ correct expression and vector notation |

(1 mark)

(b) Letting and , use a vector method to prove the theorem. (5 marks)

|  |
| --- |
| **Solution** |
| Note, for any vector : . |
| **Specific behaviours** |
| ✓ expresses and in terms of and   uses scalar product to expand sums and differences   simplifies scalar products as magnitudes   expresses in terms of sides   logical presentation of proof using correct vector notation |

Question 7 (7 marks)

Points and have position vectors , and respectively.

(a) Determine the position vector of , the midpoint of and . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates position vector |

(b) Determine the vectors and . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates vectors |

(c) Show that and are perpendicular. (2 marks)

|  |
| --- |
| **Solution** |
| Since scalar product is zero, then vectors are perpendicular. |
| **Specific behaviours** |
| ✓ calculates scalar product  ü interprets product |

(d) Hence, or otherwise, determine the area of triangle . (3 marks)

|  |
| --- |
| **Solution** |
| is base of triangle and is perpendicular height: |
| **Specific behaviours** |
| ✓ identifies base and perpendicular height  ü calculates magnitudes  ü calculates area |

Question 8 (8 marks)

(a) Show that . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses LHS as product and evaluates  ✓ expresses RHS as product and evaluates |

(b) Show that for all and that are positive integers, .

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression using factorials for LHS  ü divides fraction by  ü factors from numerator  ü expresses as RHS |

(c) Hence, or otherwise, evaluate , given that . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates appropriate method  ü correct value |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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